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HEAVY QUARK POTENTIAL IN LATTICE QCD AT FINITE TEMPERATURE [*,†]

V. BORNYAKOV [a,b,c], M. CHERNODUB [a,b], Y. KOMA [a], Y. MORI [a], Y. NAKAMURA [a], M. POLIKARPOV [b], G. SCHIERHOLZ [d], D. SIGAEV [b], A. SLAVNOV [e], H. STÜBEN [f], T. SUZUKI [a], P. UVAROV [b,e], A. VESELOV [b]

[a] *ITP, Kanazawa University, Kanazawa, 920-1192, Japan*
[b] *ITEP, B. Cheremushkinskaya 25, Moscow, 117259, Russia*
[c] *IHEP, Protvino, 124280, Russia*
[d] *NIC/DESY Zeuthen, Platanenallee 6, 15738 Zeuthen, Germany and Deutsches Elektronen-Synchrotron DESY, D-22603 Hamburg, Germany*
[e] *Steklov Mathematical Institute, Vavilova 42, 117333 Moscow, Russia*
[f] *ZIB, D-14195 Berlin, Germany*

Abstract

Results of the study of lattice QCD with two flavors of nonperturbatively improved Wilson fermions at finite temperature are presented. The transition temperature for $\frac{m_\pi}{m_\rho} \sim 0.8$ and lattice spacing $a \sim 0.12$ fm is determined. A two-exponent ansatz is successfully applied to describe the heavy quark potential in the confinement phase.

Studies of $N_f = 2$ lattice QCD at finite temperature with improved actions have provided consistent estimates of T_c [1, 2]. Still there are many sources of systematic uncertainties and new computations of T_c with different actions are useful as an additional check. To make such check we performed first large scale simulations of the nonperturbatively $O(a)$ improved Wilson fermion action at finite temperature. Other goals of our work were to study the heavy quark potential and the vacuum structure of the full QCD at $T > 0$.

We employ Wilson gauge field action and fermionic action of the same form as used by UKQCD and QCDSF collaborations [3] in $T = 0$ studies. To fix the physical scale and $\frac{m_\pi}{m_\rho}$ ratio we use their results. Our simulations were performed on $16^3 8$ lattices for two values of the lattice gauge coupling $\beta = 5.2, 5.25$.

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As numerical results show [1] both Polyakov loop and chiral condensate susceptibilities can be used to locate the transition point. We use only Polyakov loop susceptibility. We found critical temperature $T_c = 213(10)$ and $222(10)$MeV at $m_\pi/m_\rho = 0.78, 0.82$, respectively. These values are in good agreement with previous results [1] at comparable m_π/m_ρ.

To test finite size effects simulations on $24^3 \cdot 8$ lattice for $T/T_c = 0.94$ have been made. We found that results for all our observables agree with our smaller volume results within error bars. Thus finite size effects do not introduce strong systematic uncertainties in our results.

The heavy quark potential $V(r,T)$ in full QCD at non-zero temperature has been studied in [1]. It is given by $\langle L_{\vec{x}} L^\dagger_{\vec{y}} \rangle/9 = e^{-V(r,T)/T}$, where $L_{\vec{x}}$ is Polyakov loop. In the limit $|\vec{x} - \vec{y}| \to \infty$, $\langle L_{\vec{x}} L^\dagger_{\vec{y}} \rangle$ approaches the cluster value $|\langle L \rangle|^2$, where $|\langle L \rangle|^2 \neq 0$ because the global Z_3 symmetry is broken by the fermions.

The spectral representation for the Polyakov loop correlator is [4]

$$\langle L_{\vec{x}} L^\dagger_{\vec{y}} \rangle = \sum_{n=0}^{\infty} w_n e^{-E_n(r)/T}.$$

At $T = 0$ one gets $V(r, T = 0) = E_0(r)$. In contrast, $V(r,T)$ at $T > 0$ gets contributions from all possible states. We assume that in the confinement phase, at temperatures below T_c, the Polyakov loop correlator can be described with the help of two states, namely string state and broken string (two static-light meson) state:

$$\frac{1}{9}\langle L_{\vec{x}} L^\dagger_{\vec{y}} \rangle = e^{-(V_0 + V_{str}(r,T))/T} + e^{-2E(T)/T}\,, \tag{1}$$

$$V_{str}(r,T) = \frac{1}{6r}\mathrm{arctan}x - \frac{\pi}{12r} + \sigma(T)r + \frac{xT}{3}\mathrm{arctan}\frac{1}{x} + \frac{T}{2}\ln\Big(1 + x^2\Big), \tag{2}$$

$$E(T) = V_0/2 + m(T)\,, \tag{3}$$

where $m(T)$ is the effective quark mass at finite temperature, $x = 2rT$. The $T \neq 0$ string potential (2) was derived in [5]. The alternative fit of our data can be done using the finite temperature QCD static potential [6]:

$$V_{KMS}(r,T) = \frac{\tilde{\sigma}}{\mu}(1 - e^{-\mu r}) - \frac{\alpha}{r} e^{-\mu r}\,, \tag{4}$$

where $\tilde{\sigma}$, μ and α are parameters. We used function (4) to fit the data.

In computation of the Polyakov loops correlator to reduce statistical errors hypercubic blocking [7] has been employed. Details of this computation were reported in [8]. Parameters of the fit (1)-(3) are presented in Fig. 1. The values for the ratio $\sigma(T)/\sigma(0)$ are higher than those obtained in quenched QCD [9], especially close to T_c. The values for $m(T)$ are also 20-30 % higher than those obtained in [10]. Using parameters of the potential we calculate the string breaking distance r_{sb} from relation $V_{str}(r_{sb}, T) = 2m(T)$. In Fig. 1 one can see that r_{sb} decreases down to values ~ 0.3 fm when temperature approaches critical

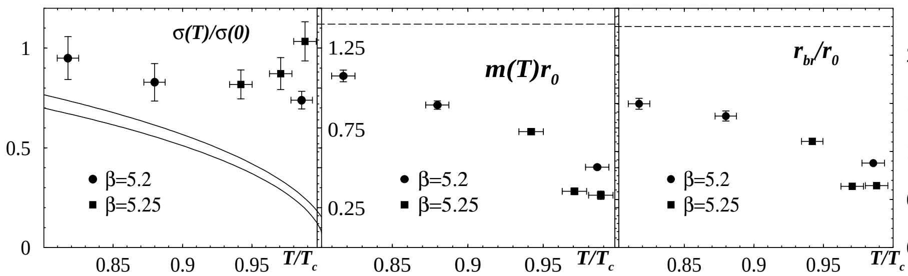

Figure 1: Best fit parameters for fit eq.(1) as functions of temperature. Solid line on the left-hand figure show quenched results[9]. Dashed horizontal lines show $T = 0$ results. $r_0 = 0.5$ fm.

value. Our fit using $V_{str}(r, T)$, eq.(2), is probably not valid when r_{sb} becomes so small. It still provides reasonable values for string tension and effective mass for $T/T_c < 0.95$ when $r_{sb} > 0.5fm$. The comparison of two fits, eqs.(1–3) and eq.(4) showed that both fits are equally good within our error bars. There is an indication that with more precise data one can discriminate between these two fits at low temperatures. We found also that parameters of the fit eq.(4) are in a clear disagreement with parameters suggested in [11].

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